Fluid and kinetic plasma instabilities in Hall effect thrusters

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Partially magnetized plasmas are unique as the magnetized electrons and nonmagnetized ions can generated anisotropic flow, primarily due to the $\mathbf{E} \times \mathbf{B}$ and diamagnetic drifts. The discharge plasma in Hall effect thrusters is complex, exhibiting nonlinear coupling of a variety of physical processes, including plasma-wall interactions, ionization, ion acceleration, and guiding center motion of electrons. The multiscale nature of the discharge plasma due to the presence of ions, electrons, and neutral atoms leads to both low- and high-frequency plasma oscillations. The low-frequency oscillations include breathing mode and azimuthally rotating spokes. Recent investigations have suggested the presence of high-frequency plasma oscillations, which are driven by kinetic instabilities, such as the electron cyclotron drift instability, modified two-stream instability, and ion-ion two-stream instability. The multidimensional plasma waves can transport and diffuse the electrons across magnetic field lines. While the theories of individual instabilities are well established, multiple instabilities are likely to occur simultaneously in the plasma devices, leading to the need for theoretical and computational framework to capture the coupling of instabilities and high-fidelity physics-based models.

I. Introduction

Low-temperature magnetized plasmas are observed widely in nature and engineering devices, including the ionosphere in space plasmas, collisionless shocks in astrophysics, Hall effect thrusters (HETs), magnetron discharges, inductively coupled plasmas (ICPs), and dusty plasmas. Similar to low-temperature plasmas, the electron temperature is typically in the range of a few to tens of electron-volts, while the ions are close to room temperature, which leads to a weakly and partially ionized plasma. The electron transport across the magnetic field lines is particularly affected, since the electrons are trapped around the magnetic field. The presence of a magnetic field can generate a variety of electron drifts, which transfer the energy and momentum to different directions and can sometimes cause different instabilities.

One of the key physical phenomena that are poorly understood in such low-temperature magnetized plasmas is the *anomalous electron transport across the magnetic field lines*. For this, many studies have relied on empirical electron mobility models (cf. two-region [1] and three-region [2] models) using drift-diffusion approximations. However, recent experimental studies indicate that the cross-field electron transport is time-dependent [3] and can be affected by the plasma turbulence driven by kinetic microinstabilities [4]. To understand such physical processes, it is critical to investigate the effects of (i) unsteady electron dynamics ($\partial/\partial t \neq 0$), (ii) inertia terms (which are the most important terms when analyzing fluid turbulence), and (iii) anisotropic pressures due to the non-Maxwellian effects.

Experimental characterization of low-temperature partially magnetized plasmas has led to new findings and indications in cross-field plasma transport. Low-frequency ionization oscillations have been investigated for HETs using ultra-fast cameras [5], high-speed Langmuir probes [6, 7], laser-induced fluorescence [8–11], and segmented anode [12].

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Recent advancement of laser Thompson scattering is promising in that high-frequency plasma wave fluctuations, e.g., ion acoustic turbulence in the MHz range, are directly measured [13]. Measurements in the hollow cathode plume detected ion acoustic waves that may cause anomalous electron transport [14, 15]. Self-organized coherent structures, namely, rotating spokes, are investigated in magnetron devices using streak cameras [16]. Advancements in experimental techniques stimulate the need for a higher-fidelity model for validating measurements and developing efficient predictive modeling capabilities.

In this paper, we summarize a few selected instabilities that arise from both kinetic and fluid descriptions. The linear instability theory can be useful for verification of the physics-based models, which can further help understand the nonlinear saturation and coupling of various instabilities, leading to plasma waves and turbulence that may contribute to anomalous electron transport across the magnetic field lines.

II. Governing equations

A. Poisson equation

Considering that the induced magnetic field is small, which is valid when the current density in the system is negligible, the plasma can be considered to be electrostatic. Thus, the Maxwell equations can be reduced to the Poisson equation, which is given by,

$$\epsilon_0 \nabla^2 \phi = -\left(\sum_{s'} q_{s'} n_{i,s'} - e n_e \right), \tag{1}$$

where ϵ_0 is the vacuum permittivity, ϕ is the electrostatic potential, $n_{i,s'}$ and $q_{s'}$ are the number density and the charge of an ion species s', e is the elementary charge, and n_e is the electron density.

B. Kinetic equations

The kinetic equation for gas species can be written as follows.

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \vec{\nabla} f + \frac{\vec{F}}{m} \cdot \vec{\nabla}_v f = \left(\frac{\delta f}{\delta t} \right)_{coll}, \tag{2}$$

where f is the velocity distribution function (VDF), $\vec{v}$ is the particle velocity, $\vec{F}$ is the force exerted onto particles, and the right hand side is the collisional term. For non-relativistic particles (which is the case for low-temperature plasmas), the force can be written as $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$, where $\vec{E}$ is the electric field and $\vec{B}$ is the magnetic field.

C. Fluid equations

1. Conservation of mass

Conservation of mass can be constructed by taking the zeroth moment of the kinetic equation. Thus, the continuity equation can be written as

$$\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s \mathbf{u}_s) = S, \tag{3}$$

where n_s is the number density, $\mathbf{u}_s$ is the bulk velocity for species s, and S is the source and sink (e.g., ionization and recombination).

2. Conservation of momentum

The equation for the fluid momentum can be formulated by taking the first moment of the kinetic equation, which can be written using conservative or primitive variables. Assuming that the distribution function is close to an isotropic Maxwellian distribution function, the conservation of momentum can be written as,

$$\frac{\partial (m_s n_s \mathbf{u}_s)}{\partial t} + \nabla \cdot \left(m_s n_s \mathbf{u}_s \mathbf{u_s} + \bar{\bar{p}}_s \right) = q_s n_s (\mathbf{E} + \mathbf{u}_s \times \mathbf{B}) + \mathbf{R}_s, \tag{4}$$

where m_s is the mass, $\bar{\bar{p}}_s$ is the pressure tensor, q_s is the charge, **E** is the electric field, **B** is the magnetic field, and **R** is the collisional drag. Here, the pressure tensor is defined using the index notation as

$$p_{ij} = mn \iiint (v_i - u_i)(v_j - u_j)\hat{f}(\vec{v})d^3\vec{v}. \tag{5}$$

Using the continuity equation, as shown in Eq. (3), and assuming the pressure tensor reduces to an isotropic pressure, the momentum equation can also be given, using the primitive variables, by:

$$\frac{\partial \mathbf{u}_s}{\partial t} + (\mathbf{u}_s \cdot \nabla)\mathbf{u}_s = -\frac{\nabla p_s}{m_s n_s} + \frac{q_s}{m_s}(\mathbf{E} + \mathbf{u_s} \times \mathbf{B}) + \frac{\mathbf{R}'_s}{m_s n_s}, \tag{6}$$

where $\mathbf{R}'$ is the modified collisional drag, which needs to account for the momentum transfer due to the source and sink in the conservation of mass. Note that the pressure is a scalar term, which is valid when the velocity distribution function (VDF) is close to an isotropic Maxwellian distribution function, i.e., the temperatures in three directions are equal. Under this condition, the pressure can be written using the ideal gas law: $p_s = n_s k_B T_s$, where k_B is the Boltzmann constant and T_s is the temperature for species s.

3. Drift-diffusion approximation

Assuming an isothermal flow and isotropic pressure, the drift-diffusion (DD) model for electrons is given by

$$\mathbf{\Gamma} \equiv n\mathbf{u} = -\underbrace{n\bar{\bar{\mu}} \cdot \mathbf{E}}_{\text{drift}} - \underbrace{\bar{\bar{D}} \cdot \nabla n}_{\text{diffusion}}, \tag{7}$$

where **u** is the bulk velocity, n is the number density, **E** is the electric field, and $\bar{\bar{\mu}}$ and $\bar{\bar{D}}$ are the mobility and diffusion coefficients, respectively. These transport coefficients become a tensor in the presence of magnetic fields, because the transport along and across the magnetic fields can differ on the order of Ω^2, where Ω is the Hall parameter, which typically has a maximum value of about 100 in the HET discharge plasma. The Hall parameter is a measure of how much the charged particles are magnetized, $\Omega = \omega_c/\nu_m$, where $\omega_{ce} = eB/m_e$ is the gyrofrequency, ν_m is the momentum transfer collision frequency, e is the elementary charge, B is the magnetic field strength, and m is the mass. Cylindrical and field-aligned coordinate systems are shown in Fig. 1. Typically in HET discharges, B_θ is assumed to be negligible, so two dimensions (r and z) are taken into account. Therefore, in the field-aligned coordinate system,

$$\bar{\bar{\mu}} = \begin{bmatrix} \mu_\parallel & 0 \\ 0 & \mu_\perp \end{bmatrix} \quad \text{and} \quad \bar{\bar{D}} = \begin{bmatrix} D_\parallel & 0 \\ 0 & D_\perp \end{bmatrix}, \tag{8}$$

where $\mu_\parallel = \mu_0 = e/m\nu_m$ and $\mu_\perp = \mu_0/(1 + \Omega^2)$ are the electron mobilities along and across magnetic field lines (*classical theory*). For cylindrical coordinates, a rotation matrix based on the angle ϕ will be applied. For low

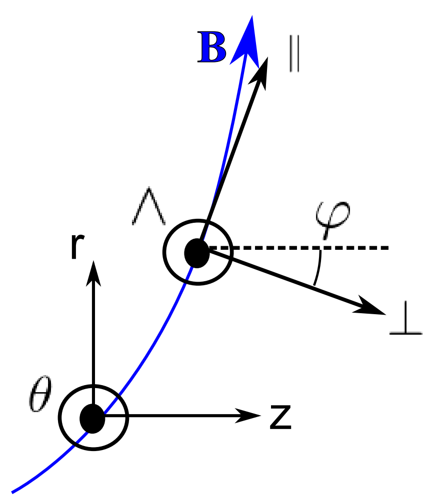

Fig. 1 Cylindrical ($r - z - \theta$) vs. Field-aligned ($\parallel - \perp - \wedge$) coordinate systems.

temperature plasma simulations, it is common to formulate a rate equation for each species combining Eqs. 3 and 7. The Scharfetter-Gummel scheme [17] is typically used to achieve stable calculations, as the first order approach is numerically unstable, particularly when the grid size is large (with respect to the Debye length).

4. Conservation of energy

Taking the second moment of the kinetic equation, as shown in Eq. (2), the conservation of total energy can be derived as,

$$\frac{\partial(n_s\varepsilon_s)}{\partial t} + \nabla\cdot\left(n_s\mathbf{u}_s\varepsilon_s + \mathbf{u}_s\cdot\bar{\bar{p}}_s + \mathbf{q}_s\right) = \mathbf{j}_s\cdot\mathbf{E} + S_\varepsilon, \tag{9}$$

where ε_s is the total energy, $\mathbf{q}_s$ is the heat flux, $\mathbf{j}$ is the current density, and S_ε is the energy exchange with different species. Here, $\varepsilon = e + K$, where e is the internal energy and $K = \frac{1}{2}m|\mathbf{u}|^2$ is the drift energy. For monatomic gases, $e = \frac{3}{2}k_BT$, where k_B is the Boltzmann constant and T is the temperature.

Using the conservation of momentum, assuming that the pressure tensor reduces to an isotropic pressure, and considering that the source term in the conservation of mass $S = n_s\nu_{\text{ion}}$, where ν_{ion} is the ionization frequency, one can derive the transport equation for the drift energy of the fluid element:

$$\frac{\partial(n_sK_s)}{\partial t} + \nabla\cdot(n_s\mathbf{u}_sK_s) = \mathbf{j}_s\cdot\mathbf{E} + \mathbf{u}_s\cdot(\mathbf{R}_s - \nabla p_s) - n_sK_s\nu_{\text{ion}}. \tag{10}$$

Similarly, one can derive the transport equation for the internal energy, which can be written as

$$\frac{\partial(n_se_s)}{\partial t} + \nabla\cdot\left[(n_se_s + p_s)\mathbf{u}_s + \mathbf{q}_s\right] = -\mathbf{u}_s\cdot(\mathbf{R}_s - \nabla p_s) + n_sK_s\nu_{\text{ion}} + S_\varepsilon. \tag{11}$$

It can be seen that combining Eqs. (10) and (11) leads to the conservation of total energy, shown in Eq. (9). Additionally, the drift-diffusion approximation can be retained when $K_s \approx 0$ in Eq. (10) [18].

III. Linear perturbation theory

Under the linear perturbation analysis, a plasma property Q can be described as a sum of the steady-state quantity and a linear perturbation, such that

$$Q = Q_0 + Q_1\exp(-i\omega t + i\mathbf{k}\cdot\mathbf{x}), \tag{12}$$

where Q_0 and Q_1 are the equilibrium (steady-state) and first-order perturbation terms of a plasma property Q, respectively, ω is the frequency, t is time, $\mathbf{k}$ is the wave vector, and $\mathbf{x}$ is the position. Here, $\omega = \omega_r + i\gamma$, where ω_r is the real frequency and γ is the imaginary part which corresponds to the growth rate.

From the Poisson equation, as shown in Eq. (1), the dispersion relation of a kinetic instability can be written as

$$\epsilon(\omega,\mathbf{k}) = 1 + \chi_i(\omega,\mathbf{k}) + \chi_e(\omega,\mathbf{k}) = 0, \tag{13}$$

where χ_i is the ion susceptibility and χ_e is the electron susceptibility. Depending on the physical processes and the governing equation of ions and electrons, a dispersion relation can be constructed.

IV. Kinetic instabilities

Collision terms are often neglected to investigate kinetic effects. The linearized Vlasov equation [19] can be given by,

$$\frac{\partial f_1}{\partial t} + \vec{v}\cdot\vec{\nabla}f_1 + \frac{q}{m}(\vec{v}\times\vec{B})\cdot\vec{\nabla}_vf_1 = -\frac{q}{m}(\vec{E}_1 + \vec{v}\times\vec{B}_1)\cdot\vec{\nabla}_vf_0. \tag{14}$$

For the resonant particles moving along $\vec{B}_0$, the $\vec{v}\times\vec{B}$ term becomes zero, which reduces to the nonmagnetized dispersion relation. However, the particles moving across $\vec{B}_0$ will be perturbed based on Eq. (14).

Consider a harmonic oscillation of electrons under an oscillating plasma wave: $\mathbf{E} = E_x\exp[i(k_xx - \omega t)]\hat{\mathbf{x}}$. Approximating the *undisturbed* orbit as $x = r_L\sin(\omega_ct)$, the equation of motion yields

$$\ddot{x} + \omega_c^2x = \frac{q}{m}E_x\exp[i(k_xr_L\sin(\omega_ct) - \omega t)] = \frac{q}{m}E_x\sum_{n=-\infty}^{\infty}\frac{J_n(k_xr_L)\exp[-i(\omega - n\omega_c)t]}{\omega_c^2 - (\omega - n\omega_c)^2}, \tag{15}$$

where $J_n(z)$ is the Bessel function of the first kind and order n.

A. Electron Bernstein mode

Solving Eq. (14), the electron susceptibility can be written as

$$\chi_e = \frac{1}{k^2\lambda_D^2}\left[1 + \frac{\omega - \mathbf{k}\cdot\mathbf{U}_d}{\sqrt{2}k_\parallel v_{th,e}} e^{-b} \sum_{n=-\infty}^{\infty} I_n(b) Z\left(\frac{\omega - \mathbf{k}\cdot\mathbf{U}_d + n\omega_{ce}}{\sqrt{2}k_\parallel v_{th,e}}\right)\right], \tag{16}$$

where $k = |\mathbf{k}| = (k_\perp^2 + k_\parallel^2)^{1/2}$, $k_\perp = (k_x^2 + k_y^2)^{1/2}$ is the wavenumber perpendicular to the magnetic field, $k_\parallel = k_z$ is the wave number parallel to the magnetic field (defining the magnetic field to be in z direction), $\lambda_D = (\epsilon_0 k_B T_e/e^2 n_0)^{1/2}$ is the Debye length, $\mathbf{U}_d = \frac{|\mathbf{E}\times\mathbf{B}|}{B^2} = -E_0/B_0\hat{\mathbf{y}}$ is the $E \times B$ drift velocity for the electrons, and $\omega_{ce} = eB_0/m_e$ is the electron cyclotron frequency. Here, T_e and T_i are the electron and ion temperatures, m_e and m_i are the electron and ion masses, n_0 is the quasineutral plasma density, B_0 is the magnitude of the applied magnetic field, $b = k_\perp^2 r_L^2$, $r_L = v_{th,e}/\omega_{ce}$ is the electron Larmor radius at the thermal velocity, Z is the plasma dispersion function given by $Z(\xi) = \pi^{-1/2}\int_{-\infty}^{\infty}\exp(-t^2)(t-\xi)^{-1}dt$, and I_n is the modified Bessel function of the first kind and order n. Further considering immobile ions, $\chi_i = 0$. The combination of Eqs. (13) and (16) assuming $\chi_i = 0$ leads to the electron Bernstein modes. The $n = 0$ mode in Eq. (16) gives rise to the Landau damping but $n \neq 0$ modes have a different type of resonance condition called the cyclotron damping, leading to heating of the electrons.

If $k_z = k_\parallel \to 0$, the electron motion parallel to the magnetic fields can be considered to be uniform. In this 2D limit, the electron susceptibility reduces to

$$\chi_e = \frac{1}{k^2\lambda_{De}^2}\left[1 - I_0(b)e^{-b} + \sum_{n=1}^{\infty}\frac{2(\omega - k_y U_d)^2 e^{-b} I_n(b)}{(n\omega_{ce})^2 - (\omega - k_y U_d)^2}\right]. \tag{17}$$

It can be seen from Eq. (17) that resonances exist for all higher harmonics $n = 1, 2, \ldots$ if $(\omega - k_y U_d)^2 = (n\omega_{ce})^2$.

B. Electron cyclotron drift instability (ECDI)

Assuming cold and nonmagnetized ions ($\omega/k \gg v_{th,i}$), the ion susceptibility term can be rewritten using the asymptotic expansion of the plasma dispersion function $Z(\xi) \approx -\xi^{-1}\left(1 + \frac{1}{2\xi^2}\right) + O(1/\xi^5)$ as:

$$\chi_i = -\frac{\omega_{pi}^2}{(\omega - \mathbf{k}\cdot\mathbf{U}_i^+)^2}, \tag{18}$$

where $\mathbf{U}_i^+$ is the bulk velocity of the ions.

Using Eqs. (16) and (18), the ECDI dispersion relation of cold, singly-charged ions and magnetized electrons in a three-dimensional configuration [20] can be written as:

$$0 = 1 - \frac{\omega_{pi}^2}{(\omega - \mathbf{k}\cdot\mathbf{U}_i^+)^2} + \frac{1}{k^2\lambda_D^2}\left[1 + \frac{\omega - \mathbf{k}\cdot\mathbf{U}_d}{\sqrt{2}k_\parallel v_{th,e}} e^{-b}\sum_{n=-\infty}^{\infty} I_n(b) Z\left(\frac{\omega - \mathbf{k}\cdot\mathbf{U}_d + n\omega_{ce}}{\sqrt{2}k_\parallel v_{th,e}}\right)\right]. \tag{19}$$

Additionally, using Eqs. (17) and (18), the 2D ECDI dispersion relation can be written as

$$0 = 1 - \frac{\omega_{pi}^2}{(\omega - \mathbf{k}\cdot\mathbf{U}_i^+)^2} + \frac{1}{k^2\lambda_{De}^2}\left[1 - I_0(b)e^{-b} + \sum_{n=1}^{\infty}\frac{2(\omega - k_y U_d)^2 e^{-b} I_n(b)}{(n\omega_{ce})^2 - (\omega - k_y U_d)^2}\right]. \tag{20}$$

While the 2D dispersion relation can be solved using a polynomial solver as can be seen from Eq. (20), the 3D dispersion relation as shown in Eq. (19) is slightly more complicated in that the plasma dispersion function needs to be solved. Ducrocq et al. has proposed a fixed-point method to solve Eq. (19), noticing that there are only two solutions to the dispersion relation and these solutions are complex conjugate, i.e., the imaginary part of the solution has the same magnitude but opposite sign [21].

C. Modified two stream instability (MTSI)

One particular instability that exists for finite $k_z\lambda_D$ is modified two-stream instability (MTSI). The dispersion relation for MTSI is derived by taking the fluid limit [22] of Eq. (19), i.e., $b = k_\perp r_L \ll 1$, and $|\omega - \mathbf{k}\cdot\mathbf{U}_d| > k_\parallel v_{th,e}$.

Here, taking the fluid limit, only the $n = 0$ term is retained:

$$1 - \frac{\omega_{pi}^2}{(\omega - \mathbf{k}\cdot\mathbf{U}_i^+)^2} + \frac{k_\perp^2}{k^2}\frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{k_\parallel^2}{k^2}\frac{\omega_{pe}^2}{(\omega - \mathbf{k}\cdot\mathbf{U}_d)^2} = 0. \tag{21}$$

It can be seen from Eq. (21) that the motion of electrons parallel to magnetic field lines is considered. It is to be noted that this dispersion relation is called the MTSI as it resembles the two-stream instability (TSI), in which two species counter stream. There are two *differences* between the MTSI and conventional TSI. (i) The third term in Eq. (21) is from the magnetized electrons but does not depend on the frequency. Hence, with the first term in Eq. (21) that comes from the Laplace operate of the Poisson equation, these two terms can be considered to be a constant value. (ii) The final term in Eq. (21) can be considered that electrons have an effective mass of $(k^2/k_\parallel^2)m_e$, which becomes large for small values of $k_\parallel$ compared to $k_\perp$, i.e., for a large wavelength mode in the parallel direction.

Unstable modes exist that are largely perpendicular to the applied magnetic field with $k_\parallel/k \sim (m_e/m_i)^{1/2}$ such that the effective electron mass is on the order of the ion mass[22]. Because of this, these waves cause electron heating parallel to $\mathbf{B}$ that is on the order of ion heating perpendicular to $\mathbf{B}$. In addition, the angle of the propagation is nearly perpendicular to the magnetic field lines as $k_\parallel < k_\perp$. In this case, $\omega_r \sim \gamma \sim \omega_{\mathrm{LH}}$, where ω_{LH} is the lower-hybrid frequency, which can be written as

$$\omega_{\mathrm{LH}} = \frac{\omega_{pi}}{(1 + \omega_{pe}^2/\omega_{ce}^2)^{1/2}}. \tag{22}$$

While this description comes from solving the Poisson equation, it is also common to make further approximations, e.g., $\omega_{pe} \gg \omega_{ce}$, leading to

$$\omega_{\mathrm{LH}} = \sqrt{\frac{m_e}{m_i}}\omega_{ce} = (\omega_{ce}\omega_{ci})^{1/2}. \tag{23}$$

It is to be noted that the lower-hybrid frequency *per se* is derived assuming cold, magnetized ions and electrons. However, the MTSI considers cold, nonmagnetized ions in addition to cold, magnetized electrons, as shown in Eq. (21). Thus, the physical meaning of the MTSI and lower-hybrid waves seems to be non-identical while the wave frequency is similar.

D. Lower hybrid drift instability (LHDI)

Electrons drifting relative to ions across a magnetic field are found to drive intermediate-frequency waves ($\omega_{pe} \gg \omega \gg \omega_{pi}$) unstable for any temperature ratio T_e/T_i. We observed that the MTSI is driven by a finite $k_\parallel (\neq 0)$ mode. However, another type of instability can be caused by the coupling of a drift wave in an inhomogeneous plasma to either the ion plasma oscillation or a lower hybrid oscillation, depending on density and field strength [23–25]. This instability is called the lower hybrid drift instability (LHDI), which was originally studied in the context of pinch devices.

Assuming $k_\parallel = 0$ and retaining only the leading order term, the dispersion relation accounts for the gradients of density, magnetic field, and electron temperature.

Two conditions of the LHDI excitation are discussed in Ref. 23. (i) When the drifts are not too weak, namely,

$$v_E v_\Delta \geq c_s^2, \tag{24}$$

where $v_E = E_0/B_0$ is the $\mathbf{E}\times\mathbf{B}$ drift,

$$v_\Delta = -\frac{k_B T_e}{m_e\omega_{ce}}\left(\frac{1}{n_e}\frac{dn_e}{dx} - \frac{1}{B}\frac{dB}{dx} - \frac{k_\perp^2 r_{Le}^2}{2}\frac{1}{T_e}\frac{dT_e}{dx}\right), \tag{25}$$

and $c_s = (k_B T_e/m_i)^{1/2}$ is the ion acoustic speed. The instability growth rate is $\gamma_{\max} = \sqrt{2}\omega_{\mathrm{LH}}$ and the wave frequency is $\omega_r = \omega_{\mathrm{LH}}$. It is to be noted that this LHDI mode will exist even if $T_e < T_i$. (ii) When the drifts are weak: $v_E v_\Delta \ll c_s^2$, the frequency and growth are given by $\omega_r = \omega_{\mathrm{LH}}$ and $\gamma_{\max} = \omega_{\mathrm{LH}}(k_y/k)(v_E v_\Delta/c_s^2)^{1/2}$, if $v_E > c_s$. Both these cases require $v_E v_\Delta > 0$ to be satisfied for the instabilities to exist. This condition resembles the gradient drift instability, which will be discussed later.

E. Ion-ion two-stream instability (IITSI) coupled with ECDI

Assuming that there is a mixture of singly and doubly charged ions, the ion susceptibility for cold, nonmagnetized ions can be written as,

$$\chi_i = -\frac{(1-\alpha)\omega_{pi}^2}{(\omega - \mathbf{k}\cdot\mathbf{U}_i^+)^2} - \frac{\alpha\omega_{pi}^2}{(\omega - \mathbf{k}\cdot\mathbf{U}_i^{2+})^2}, \tag{26}$$

where α is the ratio between the charge density of doubly charged ions and electrons and $\mathbf{U}_i^{2+}$ is the bulk velocity of doubly charged ions [4, 26].

Therefore, the 3D and 2D dispersion relation assuming a mixture of the singly and doubly charged ions, can be written using Eq. (26) as follows:

$$0 = 1 - \frac{(1-\alpha)\omega_{pi}^2}{(\omega - \mathbf{k}\cdot\mathbf{U}_i^{+})^2} - \frac{\alpha\omega_{pi}^2}{(\omega - \mathbf{k}\cdot\mathbf{U}_i^{2+})^2} + \frac{1}{k^2\lambda_D^2}\left[1 + \frac{\omega - \mathbf{k}\cdot\mathbf{U}_d}{\sqrt{2}k_{\parallel}v_{th,e}} e^{-b}\sum_{n=-\infty}^{\infty} I_n(b) Z\left(\frac{\omega - \mathbf{k}\cdot\mathbf{U}_d + n\omega_{ce}}{\sqrt{2}k_{\parallel}v_{th,e}}\right)\right] \tag{27}$$

and

$$0 = 1 - \frac{(1-\alpha)\omega_{pi}^2}{(\omega - \mathbf{k}\cdot\mathbf{U}_i^{+})^2} - \frac{\alpha\omega_{pi}^2}{(\omega - \mathbf{k}\cdot\mathbf{U}_i^{2+})^2} + \frac{1}{k^2\lambda_{De}^2}\left[1 - I_0(b)e^{-b} + \sum_{n=1}^{\infty}\frac{2(\omega - k_y U_d)^2 e^{-b} I_n(b)}{(n\omega_{ce})^2 - (\omega - k_y U_d)^2}\right]. \tag{28}$$

As the susceptibility of cold ions is written using polynomials, the 2D dispersion relation as shown in Eq. (28) can be calculated using a polynomial solver [26]. However, when applying the fixed-point solver that Ducrocq et al. proposed for Eq. (19) fails to work for Eq. (27). The reason is because there are now *four* roots for the ECDI-IITSI case instead of *two* roots for the ECDI case [27]. Hence, there are two unstable roots that are not necessarily complex conjugates for Eq. (27).

F. Current-carrying ion acoustic instability

The current-carrying ion-acoustic instability can be derived in the limit of zero magnetic field and singly charged ions (i.e., $\alpha = 0$). Here, let us consider an electron bulk velocity $U_e \neq 0$ and stationary ions. The dispersion relation can be written as

$$0 = 1 - \frac{Z'(\xi_i)}{2(k\lambda_{Di}^2)} - \frac{Z'(\xi_e)}{2(k\lambda_{De}^2)}, \tag{29}$$

where $\lambda_{Di} = [\epsilon_0 k_B T_i/(m_i \epsilon_0)]^{1/2}$, $\lambda_{De} = [\epsilon_0 k_B T_e/(m_e \epsilon_0)]^{1/2}$, $\xi_i = (\omega/k)/(\sqrt{2}v_{th,i})$, and $\xi_e = (\omega/k - U_e)/(\sqrt{2}v_{th,e})$. The solution to Eq. (29) can be derived in the large wavelength limit ($k\lambda_D \ll 1$) as follows.

$$\omega_r \approx +kc_s, \tag{30}$$

$$\frac{\gamma}{\omega_r} \approx \sqrt{\frac{\pi}{8}}\left[\frac{U_e}{v_{th,e}} - \sqrt{\frac{m_e}{m_i}} - \left(\frac{T_e}{T_i}\right)^{3/2}\exp\left(-\frac{T_e}{T_i}\right)\right]. \tag{31}$$

The solution suggests that the plasma wave occurs only in one direction, which is along the direction of the electron bulk velocity.

Kinetic simulations suggest that $U_e/v_{th,e}$ is a critical parameter that determines the amplitude of the plasma wave generated due to the current-carrying instability [28]. If $U_e > 1.3v_{th,e}$, the plasma instability transitions to a Buneman instability:

$$0 = 1 - \frac{\omega_{pe}^2}{(\omega - kU_e)^2)} - \frac{\omega_{pi}^2}{\omega^2}, \tag{32}$$

which is equivalent to the case where ions and electrons are both cold. The maximum growth rate obtained from the solution of Eq. (32) can be written as

$$\frac{\gamma}{\omega_{pe}} = \frac{\sqrt{3}}{2}\left(\frac{m_e}{2m_i}\right)^{1/3}, \tag{33}$$

which is obtained at the resonance condition $kU_e \approx \omega_{pe}$. Once the Buneman instability is excited, the plasma wave becomes large amplitude, thus reducing the bulk velocity of electrons significantly. This leads to the nonlinear saturation of the Buneman instability to follow an ion acoustic dispersion:

$$0 = 1 - \frac{\omega_{pi}^2}{\omega^2} - \frac{1}{(k\lambda_D)^2}, \tag{34}$$

which yields a wave frequency of

$$\omega_r = \pm\frac{kc_s}{\sqrt{1 + (k\lambda_D)^2}}, \tag{35}$$

indicating that the plasma wave is bi-directional.

V. Fluid instabilities

The geometry of the partially magnetized plasma is typically simplified assuming a slab (Cartesian) geometry. A static magnetic field in z direction, $\mathbf{B} = B_0\hat{\mathbf{z}}$, and an equilibrium electric field (applied electric field), $\mathbf{E}_0 = E_0\hat{\mathbf{x}}$, are considered. It is assumed that the equilibrium plasma is quasineutral and a plasma density gradient exists *locally* in x direction: $E_0 \neq 0$ and $dn_0/dx \neq 0$, generating $\mathbf{E}\times\mathbf{B}$ and diamagnetic drifts in $\pm y$ direction, for the equilibrium condition.

For magnetized electrons, using Eq. (6) and considering the equilibrium bulk velocity $\mathbf{u}_{e0} = (u_{e0x}, u_{e0y}, u_{e0z})^\intercal$, where subscripts x, y, and z denote the direction, the steady-state momentum equation in x and y directions can be written as,

$$0 = -\frac{k_B T_e}{m_e n_0}\frac{\partial n_0}{\partial x} - \frac{e}{m_e}(E_0 + u_{e0y}B_0), \tag{36}$$

$$0 = \frac{e}{m_e}u_{e0x}B_0, \tag{37}$$

assuming quasineutrality for the equilibrium condition.

The equilibrium density gradient and electric field are only considered in x direction. While Eq. (37) results in $u_{e0x} = 0$, Eq. (36) yields the well-known drifts:

$$u_{e0y} = -\frac{E_0}{B_0} - \frac{k_B T_e}{e n_0}\frac{n_0'}{B_0}, \tag{38}$$

where $n_0' = dn_0/dx$ is the plasma density gradient. The first term in Eq. (38) is the $\mathbf{E}\times\mathbf{B}$ drift and the second term is the diamagnetic drift, which can be written as u_E and u_*, respectively. The diamagnetic drift does not come from the single particle trajectory analysis but appears as an equilibrium drift from the fluid theory, while the $\mathbf{E}\times\mathbf{B}$ drift can be derived from single particle trajectories. Nonetheless, the diamagnetic drift is a steady-state bulk velocity that can propagate in the same or opposite direction of the $\mathbf{E}\times\mathbf{B}$ drift.

A. Gradient-drift instability with homogeneous magnetic field

The linear perturbation terms of the electron bulk velocity can be defied as $\mathbf{u}_{e1} = (u_{e1x}, u_{e1y}, 0)^\intercal$ for magnetized electrons. Consider that the static magnetic field is uniform in space, i.e., $\vec{B}$ is constant.

The linear perturbation of conservation of mass can be derived as,

$$\frac{\partial n_{e1}}{\partial t} + n_0\nabla\cdot\mathbf{u}_{e1} + \mathbf{u}_{e0}\cdot\nabla n_{e1} + \mathbf{u}_{e1}\cdot\nabla n_0 = 0. \tag{39}$$

Thus, the perturbed electron density can be written as,

$$n_{e1} = \frac{n_0 k_y u_{e1y} - i u_{e1x} n_0'}{\tilde{\omega}}, \tag{40}$$

where $\tilde{\omega} = \omega - k_y u_{e0y}$. Furthermore, if the linear perturbation is considered to be in y direction, e.g., $\phi_1 \exp(-i\omega t + i k_y y)$ for the electric field using the electrostatic assumption: $\mathbf{E} = -\nabla\phi$, the linear perturbation form of the electron bulk velocities can be derived from the conservation of momentum (without making any assumptions about the unsteady and inertia terms) [29] as

$$u_{e1x} = \frac{i}{\tilde{\omega}^2 - \omega_{ce}^2}\left[\tilde{\omega}k_n v_{th}^2\frac{n_{e1}}{n_0} - \omega_{ce}k_y\left(v_{th}^2\frac{n_{e1}}{n_0} - \frac{e}{m_e}\phi_1\right)\right], \tag{41}$$

$$u_{e1y} = \frac{1}{\tilde{\omega}^2 - \omega_{ce}^2}\left[-\omega_{ce}k_n v_{th}^2\frac{n_1}{n_0} + \tilde{\omega}k_y\left(v_{th}^2\frac{n_{e1}}{n_0} - \frac{e}{m_e}\phi_1\right)\right]. \tag{42}$$

Assuming $\tilde{\omega}^2 \ll \omega_{ce}^2$ and using Eqs. (41) and (42), Eq. (40) can be written as,

$$\frac{n_{e1}}{n_0} = \frac{e\phi_1}{m_e}\frac{k_y^2\tilde{\omega} - k_n k_y \omega_{ce}}{\omega_{ce}^2\tilde{\omega} + \left(k_y^2 + k_n^2\right)v_{th}^2\tilde{\omega} - 2k_n k_y \omega_{ce} v_{th}^2}. \tag{43}$$

Note that this equation is similar to Eq. (10) in Ref. 29, except for the coefficient of the last term in the denominator. In addition, if the bulk velocity of ions is negligible in y direction, the ion dispersion can be written as

$$\frac{n_{i1}}{n_0} = \frac{e k_y^2}{m_i \omega^2} \phi_1. \tag{44}$$

Assuming quasineutrality and perturbations occur in the azimuthal direction, which are similar assumptions employed for the Rayleigh-Taylor instability theory [19], the dispersion relation for partially magnetized plasmas can be derived as,

$$\frac{m_e k_y}{m_i \omega^2} = \frac{k_y \tilde{\omega} - k_n \omega_{ce}}{[\omega_{ce}^2 + \left(k_y^2 + k_n^2\right) v_{th}^2]\tilde{\omega} - 2 k_n k_y \omega_{ce} v_{th}^2}, \tag{45}$$

where $\tilde{\omega} = \omega - k_y u_{e0y}$ is used throughout the derivation. Here, this drift-shifted frequency can be written as $\tilde{\omega} = \omega - \omega_E - \omega_*$, where $\omega_E = k_y u_E$, $\omega_* = k_y u_*$, $u_E = -E_0/B_0$, and $u_* = -k_n k_B T_e/(eB_0)$, as can be seen from Eq. (38). It can be seen that Eq. (45) yields a third-order equation for ω, from which the damping and linear instability growth can be evaluated. The solution to Eq. (45) provides two conditions for the gradient drift instability to be unstable: (i) $\mathbf{E} \cdot \nabla n_0 > 0$, which is similar to the (modified) Simon-Hoh instability, and (ii) $\mathbf{E} \cdot \nabla n_0 < 0$ with the diamagnetic drift being sufficiently larger than the electron thermal velocity. The results of this gradient drift-instability is discussed in Ref. 30.

B. Gradient-drift instability with inhomogeneous magnetic field

To discuss the effects of the inhomogeneous magnetic fields on the fluid instabilities, one needs to utilize the $\nabla \cdot \mathbf{u}_{e1}$ term in Eq. (39). One of the strategies taken is to consider the left hand side of the electron momentum equation (unsteady and inertia terms) to be negligible [31, 32]. In addition, the perturbation terms shall be considered in 2D (in x and y). Therefore, the perturbed conservation of momentum reduces to:

$$0 = -\frac{\partial \phi_1}{\partial x} + \frac{k_B T_e}{e n_0^2}\frac{\partial n_0}{\partial x} n_{e1} - \frac{k_B T_e}{e n_0}\frac{\partial n_{e1}}{\partial x} - u_{e1y} B_0, \tag{46}$$

$$0 = -\frac{\partial \phi_1}{\partial y} - \frac{k_B T_e}{e n_0}\frac{\partial n_{e1}}{\partial y} + u_{e1x} B_0, \tag{47}$$

where $n_0 = n_0(x)$ and $B_0 = B_0(x)$, but $\phi = \phi(x, y)$, $n_{e1} = n_{e1}(x, y)$, $u_{e1x} = u_{e1x}(x, y)$, and $u_{e1y} = u_{e1y}(x, y)$. It is to be noted that this formulation *decouples* the u_{e1x} and u_{e1y}. Here,

$$\nabla \cdot \mathbf{u}_{e1} = \frac{\partial u_{e1x}}{\partial x} + \frac{\partial u_{e1y}}{\partial y} = i k_y \left(-\phi_1 \frac{e n_0}{k_B T_e} + n_{e1}\right) u_B, \tag{48}$$

where $u_B = -\frac{k_B T_e}{e B_0^2} \partial B_0/\partial x$. Using Eq. (39), this leads to

$$\frac{n_{e1}}{n_0} = \frac{\omega_* - \omega_B}{\omega - \omega_B - \omega_E} \frac{e\phi_1}{k_B T_e}, \tag{49}$$

where $\omega_B = k_y u_B$, which is due to the gradient of the magnetic field. Considering a quasineutral plasma, one can equate Eq. (44) (accounting for the perturbation in x and y) and Eq. (49) to obtain a dispersion relation:

$$\frac{k_\perp^2 c_s^2}{\omega^2} = \frac{\omega_* - \omega_B}{\omega - \omega_B - \omega_E}. \tag{50}$$

Thus, the solution of Eq. (50) can be written as,

$$\omega = \frac{k_\perp^2 c_s^2}{2(\omega_* - \omega_B)} \left[1 \pm \sqrt{1 - 4\frac{k_y^2}{k_\perp^2}\rho_i^2 \Delta}\right], \tag{51}$$

where $\rho_i = m_i c_s/(eB_0)$ is the ion Larmor radius based on the ion acoustic speed and

$$\Delta = \frac{\partial}{\partial x} \ln\left(\frac{n_0}{B_0^2}\right) \left[\frac{e E_0}{k_B T_e} + \frac{\partial}{\partial x} \ln(B_0^2)\right]. \tag{52}$$

The instability condition for the gradient drift instability assuming decoupled electron momentum can be given by

$$\frac{k_y^2}{k_\perp^2}\rho_i^2\Delta > \frac{1}{4}. \tag{53}$$

C. Resistive instability

Reference 33 discusses that both electrostatic lower-hybrid waves and electromagnetic Alfven waves transverse to the applied electric and magnetic field are found to be unstable due to collisions in the $\mathbf{E}\times\mathbf{B}$ electron flow.

The electrostatic theory is derived neglecting the pressure gradient and including a collisional drag term in the electron momentum equation. This leads to the perturbed electron density to be

$$\frac{n_{e1}}{n_0} = \left(1 - i\frac{\nu_e}{\omega - \omega_E}\right)\frac{k^2 e\phi_1}{m_e\omega_{ce}^2}, \tag{54}$$

where ν_e is the electron momentum-transfer collision frequency. Note that this equation is consistent with Eq. (43) when the pressure gradient is neglected. Combining the electrons and ions, the dispersion relation can be written as

$$1 - \frac{\omega_{pi}^2}{\omega^2} + \frac{\omega_{pe}^2}{\omega_{ce}^2}\left(1 - i\frac{\nu_e}{\omega - \omega_E}\right) = 0. \tag{55}$$

This dispersion relation can be seen as the lower-hybrid wave including the collisions. The solution to the electrostatic resistive instability can therefore be written as,

$$\omega_r \approx \pm\omega_{\mathrm{LH}}, \tag{56}$$

$$\gamma \approx \omega_{\mathrm{LH}}\frac{\nu_e}{2\omega_E}. \tag{57}$$

VI. Conclusion

The discharge plasma in Hall effect thrusters is partially magnetized, providing a unique condition for various instabilities to exist. Here, in this paper, we summarize a few selected fluid and kinetic electrostatic instabilities. The linear instability theory is useful for simulations to be verified. Investigation of the nonlinear coupling between instabilities and the effects on anomalous electron transport requires physics-based models.

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